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Frequency Distribution Examples
(See Following Pages)

25. The distribution of accidents for 84 randomly selected policies is as follows:

Number of Accidents	Number of Policies
0	32
1	26
2	12
3	7
4	4
5	2
6	1
Total	84

Which of the following models best represents these data?

(A) Negative binomial

(B) Discrete uniform

(C) Poisson

(D) Binomial

(E) Either Poisson or Binomial

$E[N] = 0 \cdot \frac{32}{84} + 1 \cdot \frac{26}{84} + \cdots = \checkmark$

$E[N^2] = 0^2 \cdot \frac{32}{84} + 1^2 \cdot \frac{26}{84} + \cdots =$

$\therefore Var(N) = E[N^2] - (E[N])^2 = \checkmark$

Compare $E[N]$ to $Var(N)$

$E[N] \approx Var(N) \implies N \sim$ Poisson

$E[N] < Var(N) \implies N \sim$ Neg. Binomial

$E[N] > Var(N) \implies N \sim$ Binomial

170. In a certain town the number of common colds an individual will get in a year follows a Poisson distribution that depends on the individual's age and smoking status. The distribution of the population and the mean number of colds are as follows:

	Proportion of population	Mean number of colds
Children	0.30	$\lambda =$ 3
Adult Non-Smokers	0.60	$\lambda =$ 1
Adult Smokers	0.10	$\lambda =$ 4

$$P_k = \frac{e^{-\lambda} \cdot \lambda^k}{k!}$$

Calculate the conditional probability that a person with exactly 3 common colds in a year is an adult smoker.

$$Pr(AS \mid N=3) = \frac{Pr(AS \cap N=3)}{Pr(N=3)} = \frac{Pr(N=3 \mid AS) \cdot Pr(AS)}{Pr(N=3)}$$

$$= \frac{e^{-4} \cdot \frac{4^3}{3!}(.1)}{e^{-4} \cdot \frac{4^3}{3!}(.1) + e^{-1} \cdot \frac{1^3}{3!}(.6) + e^{-3} \cdot \frac{3^3}{3!}(.3)}$$

(A) 0.12

(B) 0.16

(C) 0.20

(D) 0.24

(E) 0.28

171. For aggregate losses, S:

(i) The number of losses has a negative binomial distribution with mean 3 and variance 3.6.

(ii) The common distribution of the independent individual loss amounts is uniform from 0 to 20.

Calculate the 95th percentile of the distribution of S as approximated by the normal distribution.

(A) 61

(B) 63

(C) 65

(D) 67

(E) 69

287. For an aggregate loss distribution S:

(i) The number of claims has a negative binomial distribution with $r = 16$ and $\beta = 6$.

(ii) The claim amounts are uniformly distributed on the interval (0, 8).

(iii) The number of claims and claim amounts are mutually independent.

Using the normal approximation for aggregate losses, calculate the premium such that the probability that aggregate losses will exceed the premium is 5%.

(A) 500

(B) 520

(C) 540

(D) 560

(E) 580

288. The random variable N has a mixed distribution:

(i) With probability p, N has a binomial distribution with $q = 0.5$ and $m = 2$.

(ii) With probability $1 - p$, N has a binomial distribution with $q = 0.5$ and $m = 4$.

Which of the following is a correct expression for $\Pr(N = 2)$?

$\mathrm{Bin}(m, q) \Rightarrow P_k = \binom{m}{k} q^k (1-q)^{m-k}$

(A) $0.125p^2$

$Pr(N=2) = P_2$

(B) $0.375 + 0.125p$

(C) $0.375 + 0.125p^2$

$N \sim B(m=2, q=0.5) \Rightarrow P_2 = q^2 = 0.25$

(D) $0.375 - 0.125p^2$

$N \sim B(m=4, q=0.5) \Rightarrow P_2 = \binom{4}{2} q^2 (1-q)^2 = 6(.5)^4 = .375$

(E) $0.375 - 0.125p$

$\therefore P_2 = p \cdot (0.25) + (1-p) \cdot (.375)$
$= .375 - .125p$

$P(N=2) = E[Pr(N=2 \mid I)]$

$Pr(N=2 \mid I) = Pr(N \mid I) = 2)$	Pr
0.25	p
0.375	$(1-p)$

113. The number of claims, N, made on an insurance portfolio follows the following distribution:

n	$\Pr(N = n)$
0	0.7
2	0.2
3	0.1

If a claim occurs, the benefit is 0 or 10 with probability 0.8 and 0.2, respectively.

The number of claims and the benefit for each claim are independent.

Calculate the probability that aggregate benefits will exceed expected benefits by more than 2 standard deviations.

(A) 0.02

(B) 0.05

(C) 0.07

(D) 0.09

(E) 0.12

114. A claim count distribution can be expressed as a mixed Poisson distribution. The mean of the Poisson distribution is uniformly distributed over the interval [0, 5].

Calculate the probability that there are 2 or more claims.

$P = 1 - P_0 - P_1$

(A) 0.61

$N|\Lambda \sim P(\Lambda)$ & $\Lambda \sim U(0,5)$ $\hookrightarrow f_\Lambda(\lambda) = \frac{1}{5}$

(B) 0.66

$\Pr(N=0) = E[\underbrace{\Pr(N=0|\Lambda)}_{= e^{-\Lambda}\cdot\frac{\Lambda^0}{0!} = e^{-\Lambda}}]$

(C) 0.71

(D) 0.76

(E) 0.81

$\Pr(N=0) = E[e^{-\Lambda}] = \int_0^5 e^{-t}\cdot\frac{1}{5}\,dt$

$\Pr(N=1) = E[\Pr(N=1|\Lambda)] = E[e^{-\Lambda}\cdot\Lambda]$

$= \int_0^5 t e^{-t}\cdot\frac{1}{5}\,dt$ (IBP)

130. Bob is a carnival operator of a game in which a player receives a prize worth $W = 2^N$ if the player has N successes, $N = 0, 1, 2, 3, \ldots$ Bob models the probability of success for a player as follows:

(i) N has a Poisson distribution with mean Λ. $N|\Lambda \sim P(\Lambda)$

(ii) Λ has a uniform distribution on the interval (0, 4). $\Lambda \sim U(0,4)$

Calculate $E[W]$.

$E[2^N]$ $\quad E[z^N] = P_N(z)$ pgf

For Poisson, the pgf is defined $\underline{\underline{e^{\lambda(z-1)}}}$

$= E[\underbrace{E[2^N|\Lambda]}_{\text{pgf at } z=2}]$

$= E[e^{\Lambda}] = E[g(\Lambda)]$

$= \int_0^4 e^{\lambda} \cdot \frac{1}{4}\, d\lambda = \frac{1}{4}(e^4 - 1) = 13.4$

(A) 5

(B) 7

(C) 9

(D) 11

(E) 13

131. DELETED

132. DELETED

89. You are given:

(i) Losses follow an exponential distribution with the same mean in all years.

(ii) The loss elimination ratio this year is 70%.

(iii) The ordinary deductible for the coming year is 4/3 of the current deductible.

Calculate the loss elimination ratio for the coming year.

(A) 70%

(B) 75%

(C) 80%

(D) 85%

(E) 90%

90. Actuaries have modeled auto windshield claim frequencies. They have concluded that the number of windshield claims filed per year per driver follows the Poisson distribution with parameter Λ, where Λ follows the gamma distribution with mean 3 and variance 3.

Calculate the probability that a driver selected at random will file no more than 1 windshield claim next year. $P_0 + P_1 = Pr(N=0) + Pr(N=1)$

(A) 0.15

(B) 0.19

(C) 0.20

(D) 0.24

(E) 0.31

$N|\Lambda \sim P(\Lambda)$ & $\Lambda \sim \Gamma(\alpha, \theta)$

$\Rightarrow N \sim NB(r=\alpha, \beta=\theta)$

$$\left.\begin{array}{l} E[\Lambda] = \alpha \cdot \theta \\ E[\Lambda^2] = \theta^2 \cdot (\alpha+1) \cdot \alpha \end{array}\right\} \Rightarrow Var(\Lambda) = \alpha \cdot \theta^2$$

$\alpha \cdot \theta = 3 \qquad \alpha \cdot \theta^2 = 3 \qquad \Rightarrow \theta = 1, \ \alpha = 3$

$P_0 + P_1$ using $NB(r=3, \beta=1)$

103. DELETED

104. DELETED

105. An actuary for an automobile insurance company determines that the distribution of the annual number of claims for an insured chosen at random is modeled by the negative binomial distribution with mean 0.2 and variance 0.4.

The number of claims for each individual insured has a Poisson distribution and the means of these Poisson distributions are gamma distributed over the population of insureds.

Calculate the variance of this gamma distribution.

$Var(\Lambda) = \alpha \cdot \theta^2$

(A) 0.20

(B) 0.25

(C) 0.30

(D) 0.35

(E) 0.40

$N|\Lambda \sim P(\Lambda)$ & $\Lambda \sim \Gamma(\alpha, \theta)$

$\Rightarrow N \sim NB(r = \alpha, \beta = \theta)$

$E[N] = r \cdot \beta$ $\quad Var(N) = r \cdot \beta \cdot (1+\beta)$

$r \cdot \beta = 0.2$ $\quad r \cdot \beta \cdot (1+\beta) = 0.4$

$0.2\,(1+\beta) = 0.4$

$\Rightarrow \beta = 1 \Rightarrow r = 0.2$

$\therefore \alpha = 0.2 \quad \theta = 1$

$Var(\Lambda) = 0.2 \cdot (1)^2 = 0.2$

107. For a stop-loss insurance on a three person group:

(i) Loss amounts are independent.

(ii) The distribution of loss amount for each person is:

Loss Amount	Probability
0	0.4
1	0.3
2	0.2
3	0.1

(iii) The stop-loss insurance has a deductible of 1 for the group.

Calculate the net stop-loss premium.

(A) 2.00

(B) 2.03

(C) 2.06

(D) 2.09

(E) 2.12

108. For a discrete probability distribution, you are given the recursion relation

$$\underbrace{p(k)=\frac{2}{k}p(k-1), \quad k=1,2,\ldots}$$

Calculate $p(4)$.

$\Rightarrow (a,b,0)$ $\quad \frac{P_k}{P_{k-1}} = a + \frac{b}{k}$

$a = 0 \quad b = 2$

$\Rightarrow N \sim P(\lambda = 2)$

$P_4 = \frac{e^{-2} \cdot 2^4}{4!}$

(A) 0.07

(B) 0.08

(C) 0.09

(D) 0.10

(E) 0.11

93. At the beginning of each round of a game of chance the player pays 12.5. The player then rolls one die with outcome N. The player then rolls N dice and wins an amount equal to the total of the numbers showing on the N dice. All dice have 6 sides and are fair.

Using the normal approximation, calculate the probability that a player starting with 15,000 will have at least 15,000 after 1000 rounds.

(A) 0.01

(B) 0.04

(C) 0.06

(D) 0.09

(E) 0.12

94. X is a discrete random variable with a probability function that is a member of the $(a,b,0)$ class of distributions.

You are given:

(i) $\Pr(X=0)=\Pr(X=1)=0.25$

$p_0 = p_1 = 0.25$

(ii) $\Pr(X=2)=0.1875$

$p_2 = 0.1875$

Calculate $\underbrace{\Pr(X=3)}_{p_3}$.

$$\frac{p_k}{p_{k-1}} = a + \frac{b}{k}$$

$$\frac{p_1}{p_0} = a + \frac{b}{1} \Rightarrow \frac{0.25}{0.25} = 1 = a+b$$

$$\frac{p_2}{p_1} = a + \frac{b}{2} \Rightarrow \frac{.1875}{0.25} = 0.75 = a + \frac{b}{2}$$

$$\left.\begin{array}{l} a+b=1 \\ 2a+b=1.5 \end{array}\right\} \Rightarrow \begin{array}{l} a=0.5 \\ b=0.5 \end{array}$$

$$p_3 = \left(a+\frac{b}{3}\right)\cdot p_2 = \left(0.5+\frac{0.5}{3}\right)\cdot(.1875)$$

$$= 0.125$$

(A) 0.120

(B) 0.125

(C) 0.130

(D) 0.135

(E) 0.140

165. For a collective risk model:

(i) The number of losses has a Poisson distribution with $\lambda = 2$.

(ii) The common distribution of the individual losses is:

x	$f_X(x)$
1	0.6
2	0.4

An insurance covers aggregate losses subject to a deductible of 3.

Calculate the expected aggregate payments of the insurance.

(A) 0.74

(B) 0.79

(C) 0.84

(D) 0.89

(E) 0.94

166. A discrete probability distribution has the following properties:

(i) $p_k = c\left(1 + \frac{1}{k}\right)p_{k-1}$ for $k = 1, 2, \ldots$

(ii) $p_0 = 0.5$

$\frac{P_k}{P_{k-1}} = c + \frac{c}{k} \Rightarrow (a, b, 0)$ with $a = b = c$

Calculate c.

(A) 0.06

(B) 0.13

(C) 0.29

(D) 0.35

(E) 0.40

$\therefore N \sim NB(r=2, \beta)$

$P_0 = 0.5 = (1+\beta)^{-2} \Rightarrow \beta = \sqrt{2} - 1$

$\therefore N \sim NB(r=2, \beta = \sqrt{2}-1)$

$\Rightarrow a = b = \frac{\beta}{1+\beta} = c = 0.29\ldots$

282. Aggregate losses are modeled as follows:

(i) The number of losses has a Poisson distribution with $\lambda = 3$.

(ii) The amount of each loss has a Burr distribution with $\alpha = 3, \theta = 2, \gamma = 1$.

(iii) The number of losses and the amounts of the losses are mutually independent.

Calculate the variance of aggregate losses.

(A) 12

(B) 14

(C) 16

(D) 18

(E) 20

283. The annual number of doctor visits for each individual in a family of 4 has a geometric distribution with mean 1.5. The annual numbers of visits for the family members are mutually independent. An insurance pays 100 per doctor visit beginning with the 4th visit per family.

Calculate the expected payments per year for this family.

N_i = rvr # of annual doctor visits for family member i

(A) 320

(B) 323

N = rvr # of annual doctor visits for family

(C) 326

Q: $\cancel{E[N]}$ $E[Y] = E[100 \cdot (N-3)_+]$

(D) 329

$E[(N-3)_+] = E[N] - E[N \wedge 3]$

(E) 332

$N \wedge 3$	Pr
0	p_0
1	p_1
2	p_2
3	$1-p_0-p_1-p_2$

$N \overset{\text{ind.}}{=} N_1 + N_2 + N_3 + N_4$

$N \sim NB(r=4, \beta=1.5)$

$E[N] = 4 \cdot (1.5) = 6$